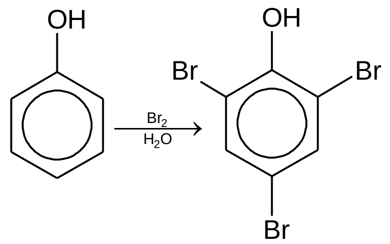
SOLUTIONS
SECTION – I: CHEMISTRY

1. $CH_3CH_2CH_2CH_2OH \xrightarrow[S_N2]{PBr_3} CH_3CH_2CH_2CH_2Br$

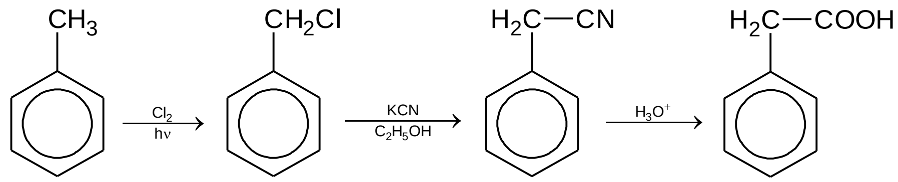
2. $CH_3-CH_2-CH_2-CH_2-\overset{\overset{O}{||}}{C}-CH_3 \xrightarrow[NaOH]{I_2} CH_3-(CH_2)_3-\overset{\overset{O}{||}}{C}-O^-Na^+ + \underset{\text{yellow}}{CHI_3}\downarrow$

3. 

4. $2.9 = \log A - \frac{Ea}{2.303R769}$

$1.1 = \log A - \frac{Ea}{2.303R667}$

$Ea = 4.17 \times 10^4$ cal mol^{-1}

5. 

6. $BeCl_2, N_3^-, N_2O, NO_2^+, O_3, SCl_2, ICl_2^-, I_3^-, XeF_2$

$BeCl_2 \longrightarrow sp \longrightarrow$ linear

$N_3^- \longrightarrow sp \longrightarrow$ linear

$N_2O \longrightarrow sp \longrightarrow$ linear

$\overset{\oplus}{N}O_2 \longrightarrow sp \longrightarrow$ linear

$O_3 \longrightarrow sp^2 \longrightarrow$ bent

$SCl_2 \longrightarrow sp^3 \longrightarrow$ bent

$I_3^- \longrightarrow sp^3d \longrightarrow$ linear

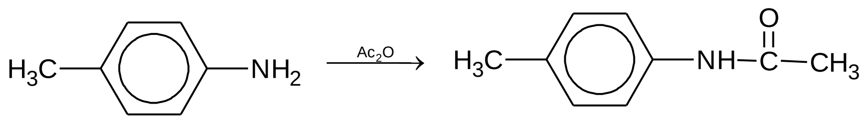
$ICl_2^- \longrightarrow sp^3d \longrightarrow$ linear

$XeF_2 \longrightarrow sp^3d \longrightarrow$ linear

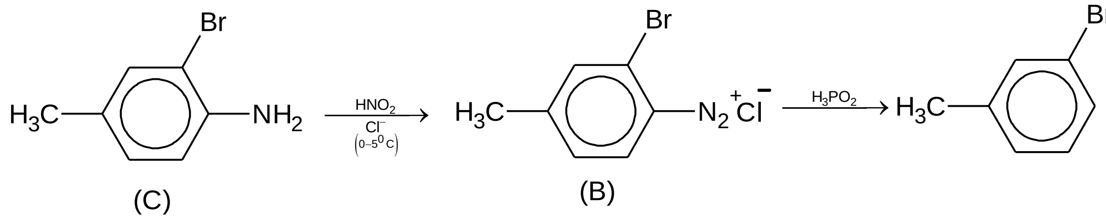
So among the following only four (4) has linear shape and no d-orbital is involved in hybridization.

7. As covalent character increases then solubility decreases

8. $C_6H_5\overset{\oplus}{N_2}\overset{\ominus}{Cl}$ gives scarlet red coloured dye with β - naphthol.

14. 

(A) (B)

15. 

SECTION – II: MATHEMATICS

PART – A

1. If a boy is selected then number of ways = $^4C_1 \cdot {}^6C_3$
If a boy is not selected then number of ways = $^6C_4$
Captain can be selected in $^4C_1$ ways
Required number of ways = $^4C_1 \cdot {}^6C_3 \cdot {}^4C_1 + {}^6C_4 \cdot {}^4C_1 = 380$

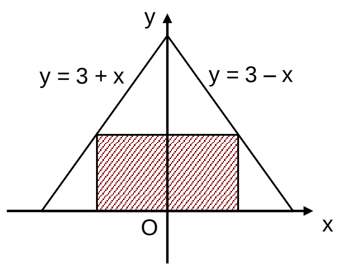
2. For max. or min. $f'(x) = 0$
$\Rightarrow x^2 - 8x + (12 + \alpha) = 0$
for one maxima and minima
$D > 0$
$\alpha < 4$

3. $(a + (b + c))^n = a^n + {}^nC_1\, a^{n-1}(b + c)^1 + {}^nC_2 a^{n-2}\,(b + c)^2 + \cdots + {}^nC_n\,(b + c)^n$.
Further expanding each term of R.H.S.,
First term on expansion gives one term.
Second term on expansion gives two terms.
Third term on expansion gives three terms and so on.
$\Rightarrow$ Total no. of terms = $1 + 2 + 3 + \cdots + (n + 1) = \frac{(n+1)\ \ (n+2)}{2}$.

4. The required numbers are 1, 2, 11, 12, 21, 22, ….., 122222222.
Let us calculate how many numbers are these.
There are 2 one-digit such numbers. There are 2^2 two-digit such numbers and so on.
There are 2^8 eight-digit such numbers. All the digit numbers beginning with 1 and written by means of 1 and 2 are smaller than 2.10^8. Thus, there are 2^8 such nine-digit numbers.
Hence the required number of numbers is
$2 + 2^2 + 2^3 + \ldots + 2^8 + 2^8 = \frac{2(2^8-1)}{2-1} + 2^8 = 2^9 - 2 + 2^8 = 766$.

5. $$\int \frac{dt}{t^2+2xt+1} = \int \frac{dt}{(t+x)^2+1-x^2} = \int \frac{dt}{(t+x)^2-(x^2-1)} \qquad \text{(since } x^2 > 1\text{)}$$
$$= \frac{1}{2\sqrt{x^2-1}}\log\left[\frac{t+x-\sqrt{x^2-1}}{t+x+\sqrt{x^2-1}}\right]+c$$

6. The area bounded by the lines
$y = 3 - |x|,\ -3 \le x \le 3$ is shown in the fig.
Area $A(x) = 2x\,.\,(3 - x)$
$\Rightarrow A'(x) = 2\,(3 - x) - 2x$
$= 6 - 4x = 0 \Rightarrow x = 3/2$
$\Rightarrow$ Maximum area of the rectangle occurs when $x = 3/2$.
Maximum area = $2.\frac{3}{2}\left(3-\frac{3}{2}\right) = \frac{9}{2}$ sq. units.



7. Let $\left(8+3\sqrt{7}\right)^{20} = I + f$, where f = fractional part and I = integral part

Also let $\left(8-3\sqrt{7}\right)^{20} = g$ then $0 < g < I$

Here $I + f + g = \left(8+3\sqrt{7}\right)^{20} + \left(8-3\sqrt{7}\right)^{20} = 2\left[8^{20} + {}^{20}C_2.8^{18}.\left(3\sqrt{7}\right)^2 + ... + {}^{20}C_{20}\left(3\sqrt{7}\right)^{20}\right]$

$\Rightarrow I + f + g$ = even integer

But $0 < f + g < 2$

So, $I + 1 =$ even Integer $(\because f + g = 1)$

$\Rightarrow I =$ odd Integer

8. $\dfrac{9! \times 2}{(3!)^3 \times 3!} = 560$

9. $F(1/x) = \int_1^{1/x} \dfrac{\ln t}{1+t+t^2}dt = \int_1^x \dfrac{\ln(1/u)}{1+\frac{1}{u}+\frac{1}{u^2}}\left(-\dfrac{du}{u^2}\right)$ $\quad\left(u = \dfrac{1}{t}\right) = \int_1^x \dfrac{\ln u}{u^2+u+1}du = F(x)$

So statement-1 is not true.

If $F(x) = \int_1^x \dfrac{\ln t}{t+1}dt$ then

$F(x) + F(1/x) = \int_1^x \dfrac{\ln t}{t+1}dt + \int_1^{1/x} \dfrac{\ln t}{t+1}dt = \int_1^x \dfrac{\ln t}{t+1}dt + \int_1^x \dfrac{\ln(1/u)}{1+\frac{1}{u}}\left(-\dfrac{du}{u^2}\right)$

$= \int_1^x \ln t\left(\dfrac{1}{t+1} + \dfrac{1}{t^2+t}\right)dt - \int_1^x \dfrac{\ln t}{t+1}\left(1+\dfrac{1}{t}\right)dt = \int_1^x \dfrac{\ln t}{t}dt = \dfrac{(\ln x)^2}{2}$

10. $f(x) = \begin{cases} \sqrt{x}\left(1 + x\sin\dfrac{1}{x}\right), & x > 0 \\ -\sqrt{-x}\left(1 + x\sin\dfrac{1}{x}\right), & x < 0 \\ 0 & x = 0 \end{cases}$

$\lim\limits_{x\to 0^-} f(x) = 0 = \lim\limits_{x\to 0^+} f(x) = f(0)$

$\therefore$ f is continuous at x = 0

$Rf'(0) = \lim\limits_{h\to 0}\dfrac{f(0+h) - f(0)}{h}$

$= \lim\limits_{h\to 0}\dfrac{\sqrt{h}\left(1 + h\sin\frac{1}{h}\right) - 0}{h}$

$= \lim\limits_{h\to 0}\left(\dfrac{1}{\sqrt{h}} + \sqrt{h}\sin\dfrac{1}{h}\right)$ does not exist

Similarly $Lf'(0)$ does not exists

$\therefore$ f(x) is continuous but not differentiable at x = 0.

11. $f(x) = \begin{cases} x[x] & ; \quad 0 \le x < 2 \\ (x-1)[x] ; & 2 \le x \le 3 \end{cases}$

$$f(x)=\begin{cases}0 & ;\quad 0\le x<1\\ x & ;\quad 1\le x<2\\ 2(x-1) & ;\quad 2\le x<3\\ 6 & ;\quad x=3\end{cases}$$

$Lf'(1) = 0$ and $Rf'(1) = 1 \Rightarrow Lf'(1) \ne Rf'(1)$
$Lf'(2) = 1$ and $Rf'(2) = 2 \Rightarrow Lf'(2) \ne Rf'(2)$
Both f′(1) and f′(2) does not exist.

12. $Lf'(0) = \lim\limits_{h\to 0}\dfrac{f(0-h)-f(0)}{-h}$

$Lf'(0) = \lim\limits_{h\to 0}\dfrac{\sin h - h - 0}{-h}$

$= 0$

13. If f(x) touches x-axis at only one irrational point, then $f(x) = (x-\alpha)^2 g(x)$, where α is irrational.
$\Rightarrow$ coefficients of f(x) can't be rational
$\Rightarrow$ for f(x) with rational coefficients, then point of touching is rational.

14. The point of touching has to be rational
$\Rightarrow$ the two roots of f(x) = 0 are rational
$\Rightarrow$ third root is also rational.

15. $f(x) = (x-\alpha)^3(x-\beta)^3$
$\Rightarrow f'(x) = (x-\alpha)^2(x-\beta)^2(2x-(\alpha+\beta))$
$\Rightarrow f''(x)$ has roots α, β and a root between $\left(\alpha, \dfrac{\alpha+\beta}{2}\right)$ and $\left(\dfrac{\alpha+\beta}{2}, \beta\right)$.

PART – B

1. (A) $\cos x + \sin^2 x$ has a local maximum at $x = \dfrac{\pi}{3}$ and $x = \dfrac{-\pi}{3}$ in the interval $(-\pi, \pi)$

(B) $\tan^{-1}(\sin x - \cos x)$ is strictly increasing in $\left(-\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right)$

(C) $x^2 - 5x + 6 = \left(x-\dfrac{5}{2}\right)^2 - \dfrac{1}{4} \ge -\dfrac{1}{4}$ for all x

$\therefore 2 < x < 3 \Rightarrow \dfrac{-1}{4} \le x^2 - 5x + 6 < 0$

$\Rightarrow \left[x^2 - 5x + 6\right] = -1$

$\therefore \int_2^3 \left[x^2 - 5x + 6\right]dx = -1$

(D) $3I = \int_{-3}^{3}\dfrac{3}{3+f(x)}dx = \int_{-3}^{3}\dfrac{3}{3+f(-x)}dx = \int_{-3}^{3}\dfrac{3f(x)}{3f(x)+9}dx$

$= \int_{-3}^{3}\dfrac{f(x)}{3+f(x)}dx = \dfrac{1}{2}\int_{-3}^{3}1.dx = 3$

$\therefore I = 1$

2. Given differential equation is $(x+y)^2\dfrac{dy}{dx} = a^2$ … (i)

Put x + y = t $\therefore 1 + \dfrac{dy}{dx} = \dfrac{dt}{dx}$, we get

$\Rightarrow t^2\left(\dfrac{dt}{dx}-1\right) = a^2 \Rightarrow \dfrac{t^2dt}{dx} = a^2 + t^2$

$\Rightarrow \dfrac{t^2dt}{a^2+t^2} = dx$

Integrating, $\int \frac{t^2}{a^2+t^2} dt = \int dx$

$\Rightarrow \int \left(1 - \frac{a^2}{a^2 - t^2}\right) dt = \int dx$

$\Rightarrow t - a \tan^{-1} \frac{t}{a} = x + c$

$\Rightarrow x + y - a \tan^{-1} \frac{x+y}{a} = x + c$

$\therefore$ $y = a \tan^{-1} \frac{x+y}{a} + c$ is the required solution.

$\therefore$ (A) $\rightarrow$ (s)

(B) $\sec^2 y \tan x \, dy = -\sec^2 x \tan y \, dx$

$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy + \int \frac{\sec^2 x}{\tan x} dx = 0$

log tan y + log tanx = logc

log tan x tany = log c

Hence tan x tan x = c is the required solution

$\therefore$ (B) $\rightarrow$ (r)

(C) $\frac{dy}{dx} = e^{3x} e^{4y}$

$\Rightarrow$ $e^{-4y} dy = e^{3x} dx + c$

$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$

Putting x = 0, we have

$-\frac{1}{4} - \frac{1}{3} = C \Rightarrow C = -\frac{7}{12}$

Hence $\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$

$\Rightarrow 7 = 3e^{-4y} + 4e^{3x}$

$\therefore$ (C) $\rightarrow$ (q)

(D) $\frac{1}{x^5 y^5} = \frac{5}{2x^2} + C$

$\therefore$ (D) $\rightarrow$ (p)

SECTION – III: PHYSICS

PART - A

1. Magnetic field at O is $\frac{\mu_0 I}{4\pi R}\left(-\vec{k}\right)+\frac{\mu_0 I}{4R}\left(-\vec{i}\right)+\frac{\mu_0 I}{4\pi R}\left(-\vec{i}\right)$

2. The magnetic dipole moment of the current carrying coil is given by

$\vec{m} = NIA\hat{n}$

$= 100 \times 0.5 \times (0.08) \times 0.04\ \hat{i} = 16 \times 10^{-2}\ Am^2\ (\hat{i})$

The torque acting on the coil is

$\vec{\tau} = \vec{m}\times\vec{B} = mB\ \ (\hat{i}\times\hat{j})$

$= 16\times10^{-2}\times\frac{0.05}{\sqrt{2}}\hat{k}$

$= 5.66 \times 10^{-3}\ (N - m)\hat{k}$.

3. Let δ_1 and δ_2 be the extensions of the two rods

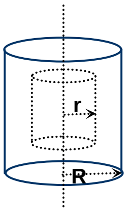
$Y_1 = \frac{F/A}{\delta_1/\ell} = \frac{F\ell}{A\delta_1}$

4. Consider a cylindrical element of radius 'r' and length 'ℓ'. According to Gauss's Law

$\int \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$

$E(2\pi r\ell) = \frac{\rho(\pi r^2\ell)}{\varepsilon_0}$

$E = \frac{\rho r}{2\varepsilon_0}$



5. Inside the cavity, B = 0

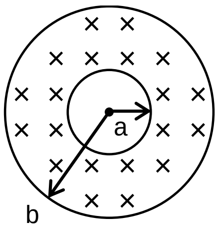
Outside the cylinder, $B = \frac{\mu_0 I}{2\pi r}$

In the shaded region

$B = \frac{\mu_0 I}{2\pi r(b^2 - a^2)}\left(r - \frac{a^2}{r}\right)$

at r = a, B = 0

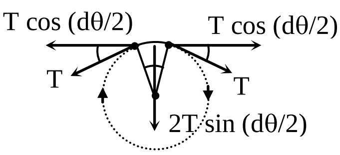
at r = b, $B = \frac{\mu_0 I}{2\pi b}$



6. $2T\sin\frac{d\theta}{2} = BiRd\theta$

$Td\theta = BiRd\theta$ (for θ small)

$T = BiR = \frac{BiL}{2\pi}$



7. Statement 2 does not confirm first statement.

8. Total energy is always negative for such systems.

9. Angular momentum is conserved. So velocity is variable.

10. Apply $\vec{F} = q\left(\vec{V}\times\vec{B}\right)$

11. –ve charge accumulates on face ABCD

12. Charge will drift due to magnetic force only.

13. Energy must be less than V_0

14. $[\alpha] = ML^{-2}T^{-2}$
Only (B) option has dimension of time
Alternatively
$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + kx^4 = kA^4$$
$$\left(\frac{dx}{dt}\right)^2 = \frac{2k}{m}\left(A^4 - x^4\right)$$
$$4\sqrt{\frac{m}{2k}}\int_0^A \frac{dx}{\sqrt{A^4 - x^4}} = \int dt = T$$
$$4\sqrt{\frac{m}{2k}}\frac{1}{A}\int_0^1 \frac{du}{\sqrt{1-u^4}} = T$$
Substitute x = Au

15. As potential energy is constant for $|x| > X_0$, the force on the particle is zero hence acceleration is zero.

PART - B

1. Uniform $\vec{E}$ constant acceleration so straight line or parabola.
Uniform $\vec{B}$ initial $\vec{v}$ along $\vec{B}$-straight line.
Uniform $\vec{B}$ initial $\vec{v} \perp \vec{B}$-circle
Uniform $\vec{B}$ initial $\vec{v} < \vec{B}$-uniform right circular cylindrical helix.
Uniform $\vec{B}$ || uniform $\vec{E}$ initial velocity along $\vec{B}$ or $\vec{E}$-straight line.
Uniform $\vec{B}$ || uniform $\vec{E}$ initial velocity $\perp$ to $\vec{B}$ or $\vec{E}$-non uniform line.
Uniform $\vec{B} \perp$ uniform $\vec{E}$ $q\vec{v} \times \vec{B} = -q\vec{E}$ straight line.

2. At steady state capacitor behave like open circuit.